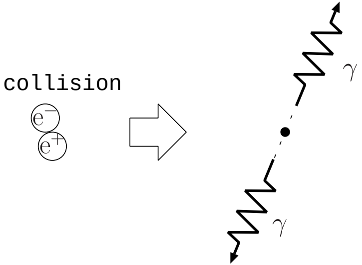
11. PET, SPECT

PET and SPECT can determine γ-ray source distribution to monitor level of biological activity.
e.g. Giving the drug which can emit γ-ray to blood vessel, we can observe amounts of blood flow.

- PET : Positron Emission Tomography
  The drug is source of positrons.
  The γ-rays are emitted by an annihilation of electron pair.
- SPECT : Single Photon Emission Computed Tomography
  The drug is source of γ-rays.

11.1 PET(Positron Emission Tomography)

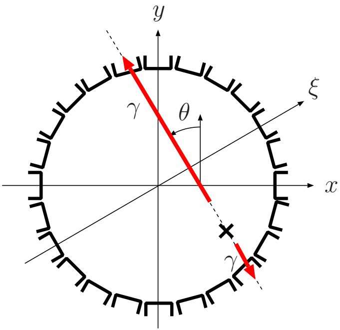
Annihilation of electron pair



$$\mathrm{e}^- + \mathrm{e}^+ \rightarrow 2\gamma(511\mathrm{keV}), \quad m_e c^2 = 511\mathrm{keV}$$

When an electron and a positron collide, two gamma-rays with 511 keV are emitted to opposite directions on a single line.
The direction of the line is random.

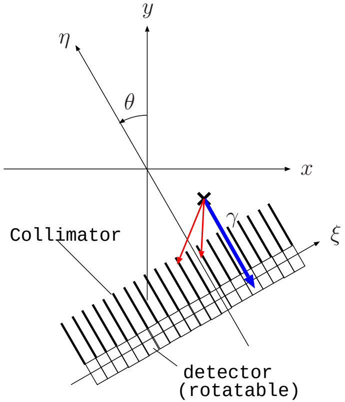
Projection data



- When annihilation occurs, two sensors located on the line passing the annihilation point detect an event at same time.
- The angle of the line corresponds to the projection angle θ in CT.
- The distance of the line from origin corresponds to the position of detector ξ in CT.
- Accumulating other events, we can obtain the statistical distribution $p(\xi, \theta)$.

The procedure to compute internal field $f(x, y)$ is same to CT.

11.2 SPCET(Single Photon Emission Computed Tomography)

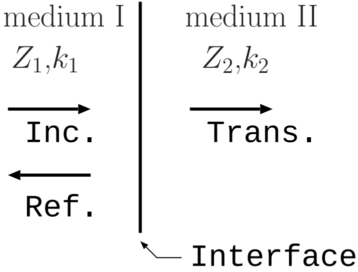


- When the array detector is aligned to perpendicular to the direction of θ, only the γ-ray with the angle θ can be detected by the detector located at ξ, since the others are shielded by the collimator.
- The projection of $p(\xi, \theta)$ corresponds to the accumulated events.

The procedure to compute internal field $f(x, y)$ is same to CT.

12. Ultrasonic Echo
12.1 Mechanism of echo

Reflection at interface

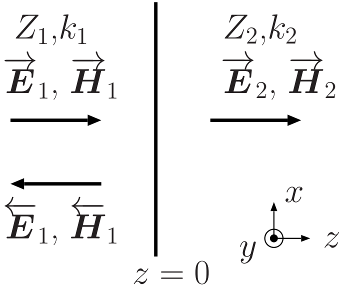


Z_i: Impedance
k_i: Wave number

The reflection occurs at the interface with impedance change.

(Proof is shown later.)

Reflection model for the electromagnetic wave



(Arrows over symbols represent propagating directions.)

- Poynting vector:

$$\overrightarrow{\boldsymbol{s}_i} = \overrightarrow{\boldsymbol{E}_i} \times \overrightarrow{\boldsymbol{H}_i} = \overrightarrow{s_i}\boldsymbol{e}_z \qquad (1)$$

$$\overleftarrow{\boldsymbol{s}_i} = \overleftarrow{\boldsymbol{E}_i} \times \overleftarrow{\boldsymbol{H}_i} = \overleftarrow{s_i}(-\boldsymbol{e}_z) \qquad (2)$$

- Definition of EM-wave:

(Definition to satisfy Eq. (1) and (2))

$$\begin{cases} \overrightarrow{\boldsymbol{E}_i} = +\overrightarrow{E_i}e^{-jk_iz}\boldsymbol{e}_x \\ \overrightarrow{\boldsymbol{H}_i} = +\overrightarrow{H_i}e^{-jk_iz}\boldsymbol{e}_y \\ \overleftarrow{\boldsymbol{E}_i} = +\overleftarrow{E_i}e^{+jk_iz}\boldsymbol{e}_x \\ \overleftarrow{\boldsymbol{H}_i} = -\overleftarrow{H_i}e^{+jk_iz}\boldsymbol{e}_y \end{cases} \qquad (3)$$

- Definition of impedance:

$$Z_i = \frac{\overrightarrow{E_i}}{\overrightarrow{H_i}} = \frac{\overleftarrow{E_i}}{\overleftarrow{H_i}} \qquad (4)$$

- Field in each medium

$$\begin{cases} \boldsymbol{E}_1 = \overrightarrow{\boldsymbol{E}_1} + \overleftarrow{\boldsymbol{E}_1}, \ \ \boldsymbol{E}_2 = \overrightarrow{\boldsymbol{E}_2}, \\ \boldsymbol{H}_1 = \overrightarrow{\boldsymbol{H}_1} + \overleftarrow{\boldsymbol{H}_1}, \ \boldsymbol{H}_2 = \overrightarrow{\boldsymbol{H}_2} \end{cases} \tag{5}$$

- Boundary condition: (Continuity of tangential components)

  At $z = 0$

$$\begin{cases} \boldsymbol{E}_1 \cdot \boldsymbol{e}_x = \boldsymbol{E}_2 \cdot \boldsymbol{e}_x \\ \boldsymbol{H}_1 \cdot \boldsymbol{e}_y = \boldsymbol{H}_2 \cdot \boldsymbol{e}_y \end{cases} \tag{6}$$

$$\Leftrightarrow \begin{cases} \overrightarrow{E_1} + \overleftarrow{E_1} = \overrightarrow{E_2} \\ \overrightarrow{H_1} - \overleftarrow{H_1} = \overrightarrow{H_2} \end{cases} \tag{7}$$

From Eq. (4) and Eq. (7)

- Reflection wave

$$\overleftarrow{E_1} = \underbrace{\frac{Z_2 - Z_1}{Z_2 + Z_1}}_{\equiv R} \overrightarrow{E_1} \tag{8}$$

- Transmission wave

$$\overrightarrow{E_2} = \frac{2Z_2}{Z_2 + Z_1} \overrightarrow{E_1} \tag{9}$$

In the case $Z_2 \neq Z_1$:

$$R = \frac{Z_2 - Z_1}{Z_2 + Z_1} \neq 0.$$

$\Rightarrow$ Reflection occurs at the interface.

12.2 Impedance

Electro-Magnetic wave	$Z = \dfrac{E\ [\mathrm{V/m}]}{H\ [\mathrm{A/m}]}$
Electronics	$Z = \dfrac{V\ [\mathrm{V}]}{I\ [\mathrm{A}]} = \dfrac{\text{Electric potential}}{\text{Electric current } (\propto \text{ velocity})}$
Fluid mechanics	$Z = \dfrac{p}{v} = \dfrac{\text{Acoustic pressure (potential)}}{\text{Acoustic velocity}}$

Impedance of fluid depends on the velocity.

Acoustic velocity and Frequency

- Acoustic velocity

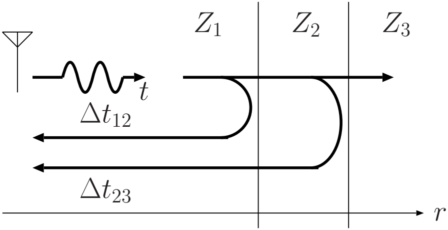
Medium	Acoustic velocity [m/s]
Air	344
Water	$\simeq$1,500
Fat	$\simeq$ 1,450
internal organs, muscle	$\simeq$ 1,550

- Frequency(at $v = 1,500$)
  $f = v/\lambda$
  λ 0.5 mm $\sim$ 0.1 mm
  f 3 MHz $\sim$ 15 MHz
  $\rightarrow$ Ultrasonic wave

12.3 Measurement system

Similarly to lader, signals are measured in polar coordinate (range and direction).

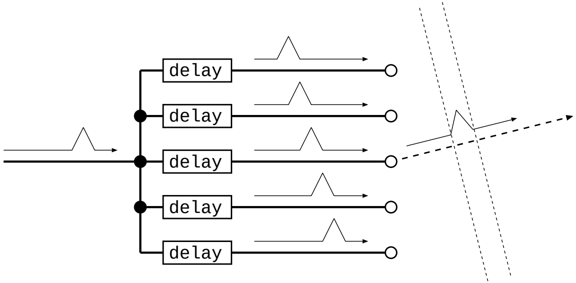
Range measurement



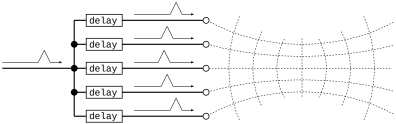
1. Emit the pulse modulated acoustic wave
2. Measure time delay of reflected pulse
3. Calculate $Z(r)$

Scanning the direction

Emitting direction can be controlled by phased array antenna:



(Non-plane wave can be formed by controlling the delays.



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