[Problem for Image Processing]
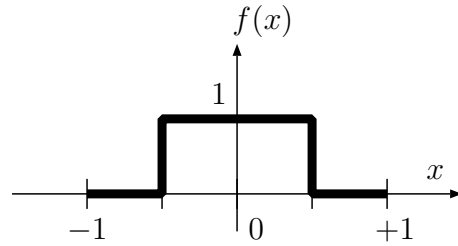
Truncation Error of Approximated Fourier Expansion

The Fourier series expansion of a function $f(x)$ in limited domain is defined by the following equation:

$$f(x) = \sum_{m=-\infty}^{\infty} F_{k_m} e^{ik_m x}, \tag{1}$$

$$F_{k_m} = \frac{1}{2l} \int_{-l}^{+l} f(x) e^{-ik_m x} \, dx, \qquad k_m = \frac{m\pi}{l}. \tag{2}$$

Express the Fourier expansion for the following function $f(x)$.
(Show F_{k_m} analytically.)

$$f(x) = \begin{cases} 1 & (|x| < 1/2), \\ 0 & (1/2 \le |x| < 1). \end{cases} \tag{3}$$

The above function is expressed with the infinite number of the terms. The approximated function $f'_M(x)$ is defined as a truncated function with the finite terms $(m = [-M, +M])$ as follows.

$$f'_M(x) = \sum_{m=-M}^{+M} F_{k_m} e^{ik_m x}. \tag{4}$$

Draw the function $f'_M(x)$ by figures for $M = 0, 1, ..., 10$.

Note to write the report:

- In the report you must show discussions. The report with only the result (equations and figure) is not acceptable. You must show that what can be understand from the results.

- You can show the description other than the given problem. (Express what you understand!)